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# Dynamics of the Vocal Fold Oscillation[1]

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**Abstract**. This paper presents an analysis of the dynamics of the vocal fold oscillation at phonation, using low dimensional mathematical models. It shows that the wavelike motion of the vocal fold mucosa is responsible for a transfer energy from the air flow to the tissues, which fuels the oscillation. A hysteresis effect is present at the onset-offset of the oscillation, commonly observed as different laryngeal configurations at the start and end of phonation. This phenomenon may be modeled as a combination of a Hopf bifurcation of subcritical type and a cyclic fold bifurcation between limit cycles. Finally, the analysis shows that smaller larynges have more restricted oscillation conditions, because they are less capable of absorbing energy from the airflow, in agreement with experimental results from woman and child voices.

## 1. Introduction

The vocal folds, together with the aerodynamics associated to the glottis and vocal tract, constitute a self-excited biomechanical oscillator that acts as the sound source during voice production. Under certain instability conditions for their biomechanical parameters such as air pressure, vocal fold tension, and glottal area, the air flow through the glottis causes the oscillation, which in turn produces the air pressure wave perceived as voice [14, 15]. Thus, it is a self-excited flow-induced oscillation, which is the same phenomenon that produces the oscillation of buildings by action of the wind, the vibration of airplane wings during flight, and the generation of sound in wind musical instruments [13].

This oscillator has a relatively complex dynamical structure, as consequence of nonlinear viscoelastic characteristics of its tissues, collisions between the opposite vocal folds, and nonlinear interaction between the airflow and the glottal area. Using mathematical models of that structure, past works have shown the existence of several nonlinear phenomena, such as multiple equilibrium positions and limit cycles [8, 3], several types of bifurcations [8, 3, 16], and chaotic behavior [3, 6]. Our recent work [8, 10, 9] has considered the dynamics of the onset and offset of the oscillation. We have shown the existence of an oscillation hysteresis phenomenon, which is commonly perceived as different laryngeal configurations at voice onset and offset. Voice onset requires a subglottal pressure above certain positive threshold level. However, after voice has started, the subglottal pressure may be decreased

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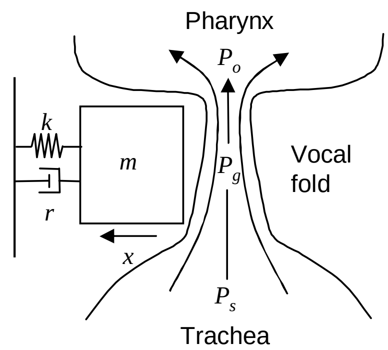


Figure 1: The one-mass model [2].

below the initial threshold, without the interruption of voice. The pressure level at which voice stops is lower than the level at which it starts. In our work, we have modeled this phenomenon as a combination of a Hopf bifurcation of the subcritical type with a cyclic fold between limit cycles. We have also applied our results to the analysis of phonation control strategies of men vs. women [9], and the development of motor control in children [10].

The following sections will present a review of the dynamics of the oscillation using low-dimensional mathematical models, and results of computer simulations.

## 2. The One-Mass Model

This was the first mathematical model for the vocal fold oscillation dynamics, proposed by Flanagan and Landgraff [2]. As shown in Figure 1, each fold is represented by a mass-damper-spring system. Both folds are assumed symmetrical, and motion is allowed only in the horizontal direction.

To simplify, we may neglect the effect of the vocal tract load on the larynx, and assume that the subglottal pressure $P_s$ is constant and the supraglottal pressure $P_o$ is equal to the atmospheric one. From experimental studies on excised larynges, we know that the vocal folds oscillate under such conditions, with similar characteristics as those observed during speech [1]. The glottal aerodynamics may be described by Bernoulli's equation, with modifications introduced by the boundary layer model of Pelorson et al. [11] for high Reynolds numbers. The resultant equation of motion is

$$m\ddot{x} + r\dot{x} + kx = dl_g P_g(x), \tag{2.1}$$

where $m$, $r$, e $k$ are positive constants that represent the mass, damping coefficient, and stiffness coefficient, respectively, of the vocal fold, $x$ is its displacement, $d$ and $l_g$ are their width and length, respectively, and $P_g(x)$ is the glottal air pressure. This pressure depends on the glottal cross-sectional area and hence is a function of the displacement $x$. It reaches its maximum when the glottis is fully closed, and decreases as the glottal area increases.

It may be easily shown that there is no limit cycle in a system represented by (2.1), which means that it is impossible to obtain a self-excited oscillation with this model. From (2.1) we obtain

$$\frac{d}{dt}(E_k + E_p) = -r\dot{x}^2 < 0,$$

where $E_k = m\dot{x}^2/2$ is the kinetic energy of the fold and $E_p = kx^2/2 + V(x)$, with $dV(x)/dx = -dl_g P_g(x)$, is its potential energy. This relation tells us that the total energy of the system decreases along trajectories, and so no oscillation of constant amplitude may exist.

However, we must note that it is possible to obtain a self-excited oscillation when the vocal tract is added to the model [14, 16]. Although this model does not capture the main oscillation mechanism of the vocal folds, nevertheless it may be used as a simple sound source in voice synthesis systems. It may be used also as a model for the falsetto register, where the vocal tract load is significant due to the high fundamental frequency of the oscillation [14].

# 3. The Mucosal Wave Model

This model by Titze [14] improves the previous mass-damper-spring system by adding a surface wave which propagates in the airflow direction. It reproduces observations of the vocal fold oscillation, which show a wavelike motion pattern of the superficial mucosal tissues [1]. Its equation of motion is [8]

$$m\ddot{x} + r\dot{x} + kx = dl_g \frac{2\tau P_s \dot{x}}{x_0 + x + \tau\dot{x}}, \tag{3.1}$$

where $\tau$ is the time delay for the mucosal wave in traveling along the glottis.

The stability of the equilibrium position at $x = 0$ may be analyzed by taking the linear part of the equation of motion in its neighborhood [12]. Linearizing (3.1), we obtain

$$m\ddot{x} + (r - 2\tau dl_g P_s/x_0)\dot{x} + kx = 0. \tag{3.2}$$

This equation shows that the airflow acts on the vocal folds as an equivalent negative dampig. When $P_s$ is zero or very small, the total damping is positive and the equilibrium position is stable. When $P_s$ is large, the total damping becomes negative, which implies a net transfer of energy from the airflow to the vocal folds. In this case, an oscillation of increasing amplitude is produced. The amplitude will be limited eventually, due to collision between the opposite vocal folds and other nonlinear effects which are not included in the model. The critical value of the subglottal pressure is the phonation threshold pressure, given by

$$P_{th} = \frac{rx_0}{2dl_g}. \tag{3.3}$$

We therefore conclude that it is the mucosal wave which allows the vocal folds to absorb energy from the airflow, permiting then their oscillation.

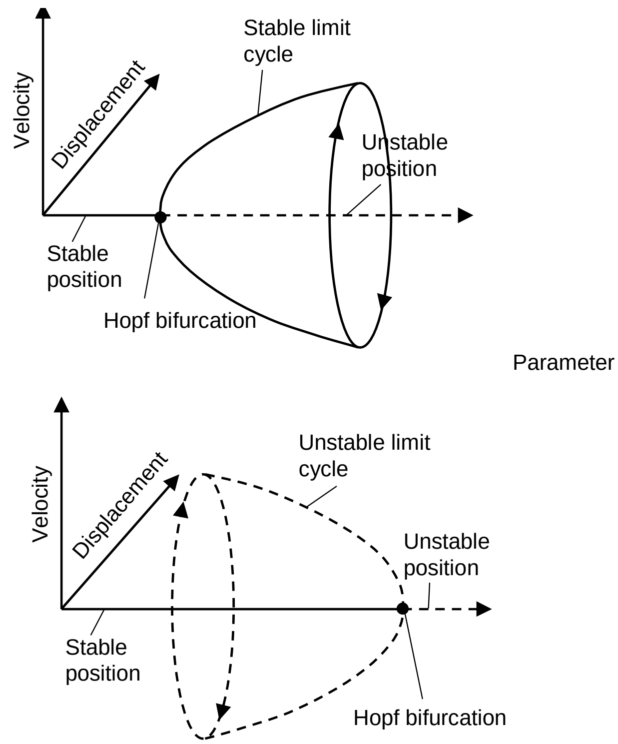


Figure 2: The Hopf bifurcation. Top: supercritical, bottom: subcritical.

## 4. Oscillation Hysteresis

Let us consider now how an oscillation may be generated from an equilibrium position. In the theory of nonlinear dynamical systems [12], the qualitative change of dynamical behavior at a critical value of a parameter is called a bifurcation. At a Hopf bifurcation, an equilibrium position changes its stability and an oscillation (limit cycle) is generated.

Two types of Hopf bifurcations are possible, as illustrated in Figure 2. In the figure, a solid line represents stable equilibrium (a position or a limit cycle), and a dashed line represents unstable equilibrium. At the supercritical Hopf bifurcation (top), as the parameter increases a stable equilibrium position bifurcates into an unstable position and a stable limit cycle. This is the simplest case, and corresponds, e.g., to the well-known van der Pol oscillator. In the subcritical Hopf bifurcation (bottom), as the parameter increases a stable equilibrium position and an unstable limit cycle coalesce into an unstable equilibrium position.

Analyzing (3.1), it can be shown that, at the phonation threshold pressure given by (3.3), two complex eigenvalues cross the imaginary axis transversally from left to right and the equilibrium position becomes unstable. At this pressure value, the equilibrium position is a weak focus and its Lyapunov number (the first nonzero

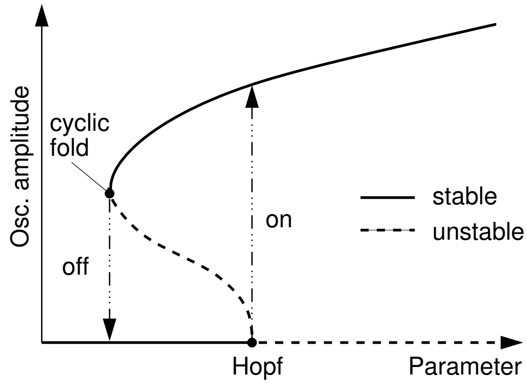


Figure 3: Oscillation hysteresis phenomenon. The curves in full and broken lines represent a stable and an unstable limit cycle, respectively. Along the horizontal axis, the full and broken line regions represent the stable and unstable regions of an equilibrium position.

derivative $d^{(k)}(0) \neq 0$ , where $d(s) = P(s) - s$ and $P(s)$ is the Poincaré map for the focus) is [8]

$$\sigma = \frac{3\pi r}{2\sqrt{mk}} \left( \frac{r\tau}{m} + \frac{3k\tau^2}{m} + 1 \right) > 0.$$

Since $\sigma > 0$, and according to Hopf Bifurcation Theorem [12], a Hopf bifurcation of the subcritical type occurs.

This type of bifurcation often appears in combination with a cyclic fold between limit cycles, as shown in Figure 3. At the cyclic fold bifurcation, the unstable limit cycle generated by the Hopf bifurcation, and a stable second limit cycle coalesce and cancel each other.

Suppose that the control parameter is increased from zero. At the Hopf bifurcation, the oscillation will start and will increase its amplitude rapidly to the stable limit cycle. If the parameter is now decreased, the oscillation will follow the curve corresponding to the stable limit cycle, until reaching the cyclic fold bifurcation. At this point, it will vanish abruptly. Thus, oscillation onset and offset occur at different threshold values of the parameter with a hysteresis effect. Note that between the onset and offset thresholds, two stable states co-exist: an equilibrium position and a stable limit cycle. This phenomenon appears commonly in cases of flow-induced oscillation [13].

A large experimental evidence shows hysteresis at the onset-offset of phonation. For example, studies of excised larynges [1] have shown that the subglottal pressure is lower at oscillation offset than at oscillation onset. Similar results have been found in subjects producing speech [4]. In all cases, the results always show that the conditions to start the vocal fold oscillation are more restricted than those to maintain it, as described above.

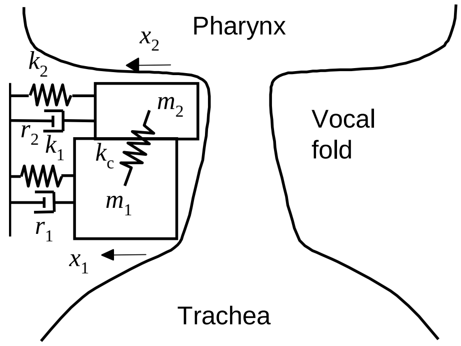


Figure 4: Two-mass model of the vocal folds [5].

## 5. The Two-Mass Model

The previous models are simple enough to permit an analytical study of their dynamics. However, to capture more details of the oscillation through computer simulations, more elaborated models are required. In increasing complexity, the next model is the popular two-mass model of Ishizaka and Flanagan [5], shown in Figure 4.

Each vocal fold is represented by a coupled pair of mass-damper-spring oscillators. When the upper mass oscillate with a phase delay in relation to the lower one, a wavelike motion in the airflow direction is reproduced. The equations of motion have the general form

$$\begin{cases} m_1\ddot{x_1} + b_1(x_1, \dot{x_1}) + s_1(x_1) + k_c(x_1 - x_2) = f_1(x_1, x_2), \\ m_2\ddot{x_2} + b_2(x_2, \dot{x_2}) + s_2(x_2) + k_c(x_2 - x_1) = f_2(x_1, x_2), \end{cases} \tag{5.1}$$

Details of these equations may be found in the cited references [3, 6, 10, 9, 11, 5].

Figure 5 shows simulation results of glottal airflow when varying the subglottal pressure from 0 to a maximum value, and back to zero. Identification of the onset threshold is in general an easy task, because the oscillation builds up quickly at that point. The offset threshold, on the other hand, is more difficult and imprecise, because the oscillation amplitude tends to vanish slowly, and it is not clear at which point the rest position has become a stable equilibrium point. However, a clear hysteresis effect may be noted: oscillation stops at a lower value of the subglottal pressure than the the value at which it starts.

Figure 6 shows plots of simulated oral airflow, as an example of the model's output. For this case, a two-tube approximation of the vocal tract in configuration for vowel /a/ [15] was added to the two-mass model.The simulations were obtained by varying the glottal half-width from 0.02 cm to 0.1 cm, and then back to the original value, following a sinusoidal pattern. This variation pattern imitates the glottal abduction-adduction gesture during the production of utterance /aha/ in running speech [10]. Oral airflow of adult man, woman and 5-year-old female child were simulated by adjusting the dimensions of the models to their respective anatomy.

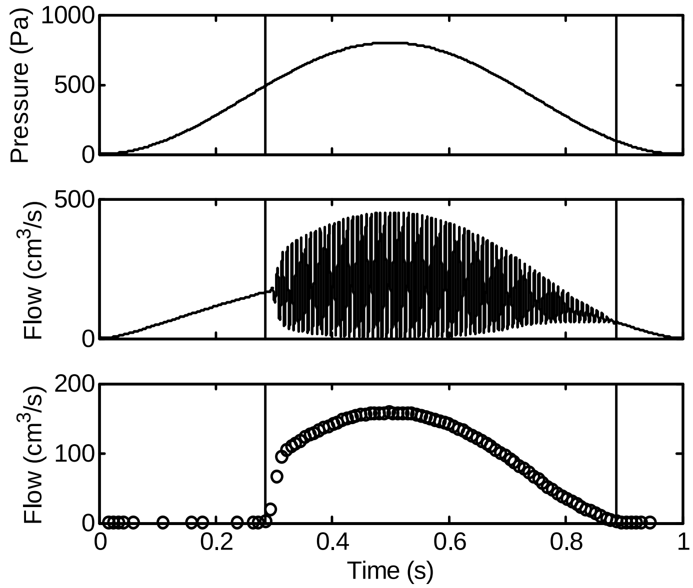


Figure 5: Simulation of glottal airflow when varying the subglottal pressure. Top: subglottal pressure, middle: glottal airflow, bottom: rms value of the AC glottal airflow. The left and right vertical lines mark the position of the oscillation onset and offset, respectively.

Comparing the plots, we see that the male flow has larger amplitude and lower fundamental frequency, as expected. In the female case, the glottal pulses stop at the peak abduction, and restart at the end of the following adduction, with a hysteresis effect. In the child case, the glottal pulses stop even earlier than the female case, at a lower value of the glottal width. The plots show that the oscillation conditions becomes more restricted as the laryngeal size decreases. Smaller larynges have more restricted phonation regions because the medial surface of the vocal folds is smaller. It is on this surface where the energy transfer from the airflow to the vocal fold oscillation takes place. In fact, (3.2) shows that the amount of energy transferred is proportional to the fold medial surface area $dl_g$. This result would explain the higher incidence of devoicing during glottal abduction-adduction for /h/ in running speech in women as compared to men [7]. As the vocal folds abduct, they easily reach the oscillation offset threshold in women, whereas men would require more extreme degrees of abduction. It would also agree with the observation of higher values of subglottal pressures in children during phonation [7], consequence of a larger value of the phonation threshold pressure.

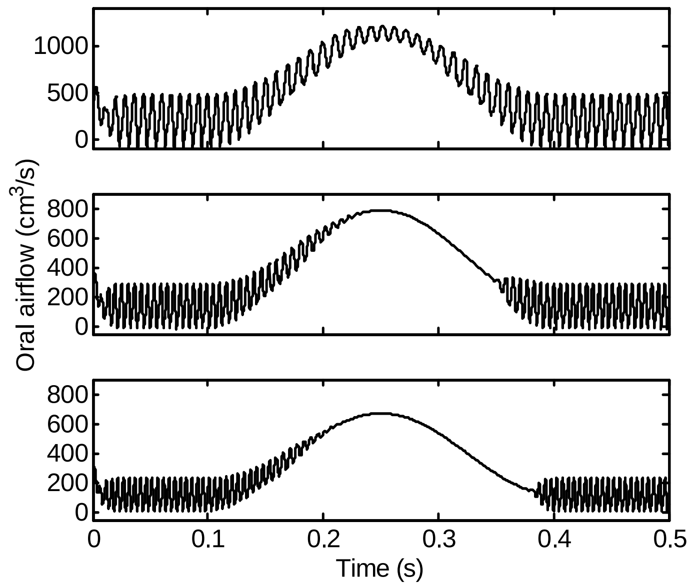


Figure 6: Oral airflow patterns during a vocal fold abduction-adduction gesture. Top: male adult, middle: female adult, bottom: 5-year-old child.

# 6. Conclusion

This paper has analyzed the dynamics of the vocal fold oscillation at phonation using low-dimensional models. It has shown that, although the main motion of the vocal folds to open and close the glottis lies in the horizontal direction (perpendicular to the airflow), it is not enough to describe the oscillatory dynamics. The wavelike motion pattern of the mucosa must be also included, which is responsible for the transfer of energy form the airflow to the vocal folds, to fuel their oscillation. The oscillation is produced when the energy absorbed form the airflow overcomes the energy dissipated in the tissues.

A hysteresis effect is always present at the oscillation onset-offset. In general, the biomechanical conditions to start the oscillation are more restricted than the conditions to maintain it after it has started. This is a common phenomenon in cases of flow-induced oscillation, and may be modeled by the combination of a subcritical Hopf bifurcation and a cyclic fold bifurcation.

It is therefore possible to capture the oscillation dynamics with simple models, as those considered here. In particular, the two-mass model seems a convenient balance between the simplicity for analytical studies of the oscillation dynamics, and the required complexity to permit simulations of phonation with realistic detail.

**Resumo**. Este artigo apresenta uma análise da dinâmica da oscilação das pregas vocais durante a fonação, utilizando modelos matemáticos de baixa dimensionalidade. Mostra-se que o movimento ondulatório da mucosa das pregas vocais é responsável pela transferência de energia do fluxo de ar aos tecidos, para alimentar a oscilação. Um efeito de histerese está presente no início e fim da oscilação, comumente observado como diferentes configurações da laringe no início e fim da fonação. Este fenômeno pode ser modelado como uma combinação de uma bifurcação de Hopf de tipo subcrítica, e uma bifurcação sela-nó entre ciclos limites. Finalmente, mostra-se que laringes pequenas possuem condições de oscilação mais restritas porque são menos capazes de absorver energia do fluxo de ar, de acordo com resultados experimentais em vozes de mulheres e crianças.

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