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Capacited Vehicle Routing Problem with CO_2 Emission Minimization Considering Path Slopes

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ABSTRACT. This work presents the application of a CO_2 emission estimation function for cargo vehicles on a Capacited Vehicle Routing Problems (CVRP) setting, considering route's slopes variation. Comparisons were established with functions minimizing fuel consumption and route length in a case study about selective collection of recyclable waste at Sorocaba, state of São Paulo, Brazil. Routes with lower emissions have been achieved without significantly increasing fuel consumption or distance traveled.

Keywords: CO_2 emission, vehicle routing problem, path slope.

1 INTRODUCTION

Carbon dioxide (CO_2) emissions from fossil fuel powered vehicles are an environmental problem because they contribute directly to the greenhouse gases (GHG) effect. Thus, minimization of such emissions can result in an overall decrease of pollution levels.

Green Vehicle Routing Problems (G-VRP) are VRP aimed for environmental problems such as fossil fuel emissions and consumption reduction or traffic jam prevention, to name but a few.

Specifically about GHG emissions, [10] compiles data about the influence of driving patterns over emission factors for heavy duty vehicles. This methodology has been applied for VRPs in [11], [8], [7] and [13].

The aforementioned driving patterns are relaxed in the form of a *Pollution-Routing Problem* (PRP) as explored by [2], [5] and [4]. They assume that the instantaneous emission rate E (g/s) at

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the exhaust is directly related to fuel consumption rate F (g/s) through the relation $E = \delta_1 F + \delta_2$ where δ_1 and δ_2 are GHC specific parameters. The exact same formulation is used by [17] on a VRP with *backhauling*.

In [22] a bi-objective Green Vehicle Routing and Scheduling Problem (G-VRSP) is presented. The main objective is the minimization of CO_2 emissions, depending on the types both of the vehicle and the fuel. The secondary objective is a penalty function accounting for delays at clients.

COPERT – *Computer programme to calculate emissions from road transport* [15] – is a software financed by the *European Environment Agency*, designed to estimate pollutant emission from road transport and urban traffic. The estimates are based on vehicle model, type of fuel, circulation area and speed limits. The fifth version of COPERT for Windows can be freely downloaded from `https://copert.emisia.com/installing/`.

In [19] and [20] a geoprocessing tool, *ArcGIS 3D*, was used to map geographical information along the routes, taking into account elevation. COPERT [15] was used to fit fuel consumption parameters. Both papers have shown that the use of geographical information (mainly elevation) can bring significant improvements in the solution when compared with "flat" models, where the effects of inclination is disregarded. Amongst all the works presented here, these two are the only ones making use of geographical information in their models. Unfortunately, ArcGIS 3D and its optimization extension *ArcGIS Network Analyst* are closed source commercial software, thus hiding the details about the VRP used.

The present work builds upon [21] where the authors propose to minimize CO_2 emissions associating them with the physical work resulting from the forces the vehicle is subjected to. Even though the authors include the slope in the equations, all results were obtained disregarding inclination (null slope). So, our contribution consists in effectively incorporating the slopes – and their corresponding effect onto CO_2 emissions – into the objective function.

A case study regarding selective collection of recyclable waste in the city of Sorocaba, state of São Paulo, Brazil, provides the common ground to compare the proposed objective function with the ones presented by [23] and applied by [18], in addition to the classical distance (symmetrical distances from the origin to depot).

The mathematical model for the Capacited Vehicle Routing Problem (CVRP) is given by (1.1), as described in [23] and [12], which in turn is an Integer Linear Programming Problem (ILPP).

Different options for the objective function (1.1a) will be examined in Section 2, so we opted to introduce the model with a generic one.

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1.1a}$$

$$\text{Subject to} \sum_{\substack{j=0 \\ i \neq j}}^{n} x_{ij} = 1 \qquad \forall i \in N^{\star} \tag{1.1b}$$

$$\sum_{\substack{j=0 \\ i \neq j}}^{n} x_{ij} - \sum_{\substack{j=0 \\ i \neq j}}^{n} x_{ji} = 0 \qquad \forall i \in N \tag{1.1c}$$

$$\sum_{\substack{j=0 \\ i \neq j}}^{n} y_{ji} - \sum_{\substack{j=0 \\ i \neq j}}^{n} y_{ij} = D_i \qquad \forall i \in N^{\star} \tag{1.1d}$$

$$y_{ij} \leq Q x_{ij}, \qquad \forall i, j \in N \tag{1.1e}$$

$$\sum_{j=0}^{n} x_{0j} = |K| \tag{1.1f}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in N^{\star}, \tag{1.1g}$$

where

- n is the number of nodes other than node 0, that represents the depot.
- $N = \{0, 1, 2, \ldots, n\}$ and $N^{\star} = \{1, 2, \ldots, n\}$.
- x_{ij} is a binary variable evaluating to 1 if the vehicle goes from i to j, 0 otherwise.
- y_{ij} is the truck load from i to j.
- c_{ij} is the cost to go from i to j.
- D_i is the demand associated with node i.
- Q is the maximum vehicle capacity.
- $|K|$ is the fleet size and also the number of routes.

Meaning of the equations in the model:

- (1.1a) Minimization of the cost to carry out the routes.
- (1.1b) Each customer can only be visited once.
- (1.1c) Vehicles must enter and leave a node right away.
- (1.1d) Represents the load increase after visiting a node. The added weight is equal to the demand of the visited node.

- (1.1e) Loads must not exceed the vehicle capacity Q. Vehicles must return to the depot when they reach or are close to its maximum capacity.
- (1.1f) Each vehicle in the fleet must follow one of the $|K|$ routes, ensuring that no cycles are formed [12].
- (1.1g) x_{ij} is a binary variable, evaluating to 1 if the vehicle goes from i to j, 0 otherwise.

Different objective functions will be plugged to the Problem (1.1), as presented below.

2 THE CVRP COST FUNCTIONS

Three cost functions for the CVRP were used, each one modeling a separate feature, namely the amount of CO_2 emission considering the slope of the streets, fuel consumption (with no slope) and distance traveled by each vehicle.

2.1 Calculation of fuel consumption and total emission

Toro et al. [21] present a Green Capacitated Location-Routing Problem (G-CLRP), where fuel consumption between nodes i and j is obtained based on the forces acting on the vehicle, as shown in Figure 1. Note that in the description of forces it is assumed that the vehicle is going up.

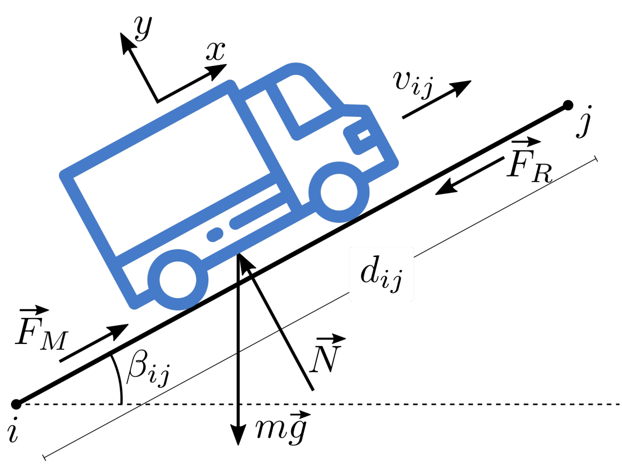


Figure 1: Forces acting on a truck moving upwards.
Source: Adapted from [21].

Defining

- β_{ij}: slope of the path between i and j.
- $\vec{F}_M$: force generated by the engine and transmitted to the tires of the vehicle.

- $m\vec{g}$: vehicle weight (mass $\times$ gravity).
- $\vec{N}$: normal force of the inclined plane on the vehicle.
- v_{ij}: vehicle speed between i and j.
- d_{ij}: distance between nodes i and j.
- $\vec{F}_R$: forces opposing to the vehicle movement (friction, wind and internal).

Force balancing equations

Assuming the same constant speed along all sections of all routes ($v_{ij} = v = \text{constant}, \forall i, j \in N$), the balance of forces is given as follows:

$$\begin{aligned} \sum \vec{F}_x = m\vec{a}_x \Rightarrow \vec{F}_M - \vec{F}_R - m\vec{g}\sin\beta_{ij} = 0 \\ \sum \vec{F}_y = m\vec{a}_y \Rightarrow \vec{N} - m\vec{g}\cos\beta_{ij} = 0 \end{aligned}$$

where

$$\vec{F}_R = \vec{F}_{R,tires} + \vec{F}_{R,w} + \vec{F}_{R,i} + \frac{mv^2}{2d_{ij}}$$

and

- $\vec{F}_{R,tires}$ represents the frictional force between tires and terrain that opposes the movement of the vehicle.
- $\vec{F}_{R,w}$ is the air resistance.
- $\vec{F}_{R,i}$ represents the equivalent force of the internal forces that oppose the movement of the vehicle.
- $\dfrac{mv_{ij}^2}{2d_{ij}}$ is the force necessary for the vehicle to reach the permanent kinetic energy regime (constant speed).
- m is the mass of the vehicle which is given by the mass of the empty vehicle m_0 plus the load t carried between nodes i and j (t_{ij}), that is, $m = m_0 + t_{ij}$.

By definition $\vec{F}_{R,tires} = \vec{N}b$, where b is a terrain-dependent constant[1]. So,

$$\begin{aligned}
\vec{F}_M &= \vec{F}_R + m\vec{g}\sin\beta_{ij} \\
&= \vec{F}_{R,\text{tires}} + \vec{F}_{R,w} + \vec{F}_{R,i} + \frac{mv^2}{2d_{ij}} + m\vec{g}\sin\beta_{ij} \\
&= \vec{N}b + \vec{F}_{R,w} + \vec{F}_{R,i} + \frac{mv^2}{2d_{ij}} + m\vec{g}\sin\beta_{ij} \\
&= (m\vec{g}\cos\beta_{ij})b + \vec{F}_{R,w} + \vec{F}_{R,i} + \frac{mv^2}{2d_{ij}} + m\vec{g}\sin\beta_{ij}.
\end{aligned}$$

Taking the magnitude of $\vec{F}_M$, we have

$$F_M = (mg\cos\beta_{ij})b + F_{R,w} + F_{R,i} + \frac{mv^2}{2d_{ij}} + mg\sin\beta_{ij}. \tag{2.1}$$

The work $U_{ij} = F_M d_{ij}$ from i to j is then given by:

$$\begin{aligned}
U_{ij} &= \left[(mg\cos\beta_{ij})b + F_{R,w} + F_{R,i} + \frac{mv^2}{2d_{ij}} + mg\sin\beta_{ij}\right]d_{ij} \\
&= \left[(m_0+t_{ij})gb\cos\beta_{ij} + F_{R,w} + F_{R,i} + \frac{(m_0+t_{ij})v^2}{2d_{ij}} + (m_0+t_{ij})g\sin\beta_{ij}\right]d_{ij} \\
&= \left[m_0 g\left(b\cos\beta_{ij} + \frac{v_{ij}^2}{2gd_{ij}} + \sin\beta_{ij}\right) + F_{R,w} + F_{R,i}\right]d_{ij} \\
&\quad + \left[g\left(b\cos\beta_{ij} + \frac{v_{ij}^2}{2gd_{ij}} + \sin\beta_{ij}\right)\right]t_{ij}d_{ij}.
\end{aligned} \tag{2.2}$$

Downward force balancing equation

What if the vehicle is going down? The forces $\vec{F}_M$ and $\vec{F}_R$ will be reversed. Consequently:

$$\begin{aligned}
\sum \vec{F}_x = m\vec{a}_x \Rightarrow \vec{F}_M - \vec{F}_R + m\vec{g}\sin\beta_{ij} = 0 \\
\sum \vec{F}_y = m\vec{a}_y \Rightarrow \vec{N} - m\vec{g}\cos\beta_{ij} = 0
\end{aligned}$$

[1] According to `https://www.ctborracha.com/borracha-sintese-historica/propriedades-das-borrachas-vulcanizadas/propriedades-tribologicas/` the coefficient of kinetic friction between the tire and the asphalt is $b = 0.72$ N. Accessed in 08/21/2019.

Similarly, we have:

$$U_{ij} = \left[m_0 g \left(b\cos\beta_{ij} + \frac{v_{ij}^2}{2gd_{ij}} - \sin\beta_{ij} \right) + F_{R,w} + F_{R,i} \right] d_{ij}$$

$$+ \left[g \left(b\cos\beta_{ij} + \frac{v_{ij}^2}{2gd_{ij}} - \sin\beta_{ij} \right) \right] t_{ij} d_{ij}.$$

General Case

Assuming constant speed, we get

$$U_{ij} = \alpha_{ij} d_{ij} + \gamma_{ij} t_{ij} d_{ij}, \tag{2.3}$$

where the constants α_{ij} depend on the mean slope between i and j, the unloaded vehicle weight, the energy to achieve the steady state speed, the resistance on the tires, the air resistance and the internal vehicle losses. Some of these quantities, in turn, depend on the speed of the vehicle. Constants γ_{ij} depend on the slope of the path between i and j and the resistance of the tires. The work required between i and j has a component that is related to the unloaded vehicle, $\alpha_{ij}d_{ij}$ and another component that is related to the carried load, that is, $\gamma_{ij}t_{ij}d_{ij}$.

The work required for the vehicle to complete a route is given by the sum of the work of each arc. Associating it to a binary variable x_{ij}, we have:

$$\sum_{i,j\in V} U_{ij} = \sum_{i,j\in V} \alpha_{ij} d_{ij} x_{ij} + \sum_{i,j\in V} \gamma_{ij} d_{ij} t_{ij}, \tag{2.4}$$

where x_{ij} is 1 if the arc between i and j is used, 0 otherwise. Note that it is possible to obtain the average slope of (i, j) thus allowing the estimation of α_{ij} and γ_{ij} for each arc, and so the objective function is linear.

The amount of fuel used to perform the total work $\sum_{i,j\in V} U_{ij}$ is obtained with a conversion factor E_1 (gallons/J). The emitted amount per unit of fuel is given by another conversion factor, E_2 (kg of CO_2/gallon). These factors depend on the type of vehicle and the fuel used, see [3] and [6]. Finally, the total emission can be calculated as:

$$E_1 \times E_2 \times \sum_{i,j\in V} U_{ij} = E \times \sum_{i,j\in V} U_{ij}. \tag{2.5}$$

Some dimensional analysis shows that

$$\underbrace{\frac{\text{gallon}}{J}}_{E_1} \times \underbrace{\frac{CO_2}{\text{gallon}}}_{E_2} \times \underbrace{J}_{\sum U_{ij}} = CO_2.$$

2.2 Fuel consumption rate – FCR

In [23], an attempt is made to solve a capacitated vehicle routing problem using a function that estimates the *fuel consumption rate* (FCR), where the distance traveled per unit volume of fuel is inversely proportional to the vehicle weight. Since Q_0 corresponds to the weight of the empty vehicle and Q_1 to the transported load, FCR is formulated as a linear function depending on Q_1:

$$\rho(Q_1) = \alpha(Q_0 + Q_1) + b, \tag{2.6}$$

with b being constant. Let Q be the maximum capacity of the vehicle, ρ^* and ρ_0 as the fuel consumption rate for the full-loaded and empty truck, respectively. It can be seen from (2.6) that $\rho^*(Q) = \alpha(Q_0 + Q) + b$ and $\rho_0 = \alpha Q_0 + b$. Then:

$$\begin{aligned}\rho^* - \rho_0 &= \alpha(Q_0 + Q) + b - (\alpha Q_0 + b)\\ &= \alpha Q_0 + \alpha Q + b - \alpha Q_0 - b\\ &= \alpha Q.\end{aligned}$$

Isolating α, we have:

$$\alpha = \frac{\rho^* - \rho_0}{Q} \tag{2.7}$$

which is the slope of the line given by Equation (2.6). Rewriting it:

$$\begin{aligned}\rho(Q_1) &= \alpha Q_0 + b + \alpha Q_1\\ &= \rho_0 + \alpha Q_1\\ &= \rho_0 + \frac{\rho^* - \rho_0}{Q} Q_1 \end{aligned} \tag{2.8}$$

where $\rho_0 = \alpha Q_0 + b \Rightarrow \rho_0 = \frac{\rho^* - \rho_0}{Q} Q_0 + b$ is the empty truck weight.

For any arc between i and j, where j is the next customer to be served after leaving i, fuel cost is given by:

$$C_{fuel}^{ij} = c_0 \rho_{ij} d_{ij}, \tag{2.9}$$

where

- c_0 is the unit cost of fuel.
- ρ_{ij} FCR along the route from i to j.
- d_{ij} distance traveled between i and j.

Denoting n as the number of customers on the route, we have:

$$C_{fuel} = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{fuel}^{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} c_0 \rho_{ij} d_{ij} x_{ij}, \tag{2.10}$$

where x_{ij} is binary, assuming value 1 if the vehicle goes from node i to j, and 0 otherwise.

Denoting y_{ij} as the load weight carried between i and j, Equation (2.8) becomes

$$\rho_{ij} = \rho_0 + \frac{\rho^* - \rho_0}{Q} y_{ij} = \rho_0 + \alpha y_{ij}. \tag{2.11}$$

Considering that vehicles have a fixed operational cost F and 0 represents the depot, the objective function becomes:

$$\min \sum_{j=1}^{n} F x_{0j} + \sum_{i=0}^{n} \sum_{j=0}^{n} c_0 d_{ij} x_{ij} (\rho_0 + \alpha y_{ij}), \tag{2.12}$$

which is nonlinear. Constraint (1.1e), however, guarantees that $y_{ij} = 0$ when $x_{ij} = 0$ [23], thus allowing us to rewrite (2.12) as

$$\min \sum_{j=1}^{n} F x_{0j} + \sum_{i=0}^{n} \sum_{j=0}^{n} c_0 d_{ij} (\rho_0 x_{ij} + \alpha y_{ij}). \tag{2.13}$$

This function was used with a CVRP by [18] in a case study applied to selective recyclable waste collection.

3 COMPUTATIONAL TESTS

Tests were performed on an Intel Core i7-2600 desktop, with 3.4 GHz, 8.0 GB of RAM and Microsoft Windows 7 Home Premium operating system. Instances were solved with CPLEX version 23.7.

Data were taken from [18] which presents a CVRP for the garbage collection of a cooperative in the city of Sorocaba, state of São Paulo, Brazil. At each node, in addition to its geographical coordinates (latitude and longitude), we add its altitude obtained with Google Earth Pro software, version 7.3.2.5776, for Debian GNU/Linux operating system. Distances between locations were calculated using the Haversine distance

$$d_{ij} = 2 \cdot 6371 \arcsin \left[\sin\left(\frac{Lat_j - Lat_i}{2}\right)^2 + \cos(Lat_i)\cos(Lat_j)\sin\left(\frac{Lon_j - Lon_i}{2}\right)^2 \right]^{1/2} \tag{3.1}$$

where *Lat* and *Lon* represent latitude and longitude, respectively [1].

Each instance was tested with the constraints presented in (1.1) and three different objective functions:

1. Distance $\sum d_{ij} x_{ij}$.
2. FCR (2.13) without the first part of the function (fixed cost of the vehicle), with the same data as modeled in [18], that is,

   $$\rho(Q) = 1 \times 10^5 Q + 0.1111,$$

   with an estimated fuel cost of R\$ 2,999 per liter (R\$: Brazilian Real).
3. Function (2.4). Description of the data follows.

Function (2.4)

In order to use (2.4), we need to determine the parameters of (2.2). We have chosen a IVECO Tector truck, 4×2, of 9 tonnes[2].

As the $F_{R,i}$ value is not available in the literature and we are not able to properly define a value for it, we are assuming $F_{R,i} = 0$. For $F_{R,w}$ (aerodynamic drag coefficient), we have:

$$F_{R,w} = \frac{1}{2}\rho C_x A v^2$$

being:

- $\rho = 1.184$ kg/m^3, air density with a temperature around 25°C.
- $C_x = 0.9$, aerodynamic coefficient for a truck.
- $A = 2.491 \times 1.890 = 4.70799$ m^2, estimated cross-sectional area for a 9-ton Tector truck.
- $v = 20 \times (1000/3600)$ m/s, constant speed between nodes.

Other function parameters:

- $m_0 = 3025$ kg, empty truck weight.
- $g = 9.81$ m/s^2, gravitational constant.
- $b = 0.72$ N, friction of the tire with the asphalt.

The angle β_{ij} can be obtained with the help of Figure 2.

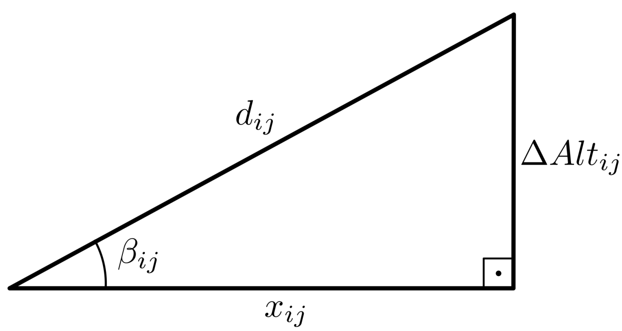


Figure 2: Right triangle used to calculate β_{ij}.

Given the distances we estimate the difference in height between two nodes:

$$\Delta Alt_{ij} = Alt_j - Alt_i,$$

[2] https://www.iveco.com/brasil/produtos/pages/tector_carac_bene.aspx

where Alt_i and Alt_j are the heights at nodes i and j, respectively, on the opposite side to β_{ij}. The adjacent side to β_{ij} is given by:

$$x_{ij} = \sqrt{d_{ij}^2 - \Delta Alt_{ij}^2}.$$

Finally,

$$\beta_{ij} = \arctan\left(\frac{\Delta Alt_{ij}}{x_{ij}}\right).$$

For the Emission Factor, we assume $E = 694$ CO_2 (g/KWh)[3], for medium-sized trucks.

3.1 Validation problem

For model validation, five points were chosen in the city of Sorocaba, state of São Paulo, Brazil, corresponding to recyclable waste generators of reasonable size (three universities, one shopping mall and one residential building playing depot). All points have different heights, as can be seen in Table 1.

Table 1: Test data.

	Height (m)	Latitude (°)	Longitude (°)	Demand (kg)
0	601	−23.50874	−47.46598	3700
1	609	−23.50325	−47.46365	3700
2	613	−23.47935	−47.41724	3700
3	667	−23.58126	−47.52405	3700
4	646	−23.53465	−47.46277	3700

Inclinations can be seen on (3.2), where we assign index 0 (origin/depot) to the first row and first column.

$$\begin{bmatrix} 0 & 0.0122 & 0.0020 & 0.0066 & 0.0155 \\ -0.0122 & 0 & 0.0007 & 0.0055 & 0.0106 \\ -0.0020 & -0.0007 & 0 & 0.0034 & 0.0043 \\ -0.0066 & -0.0055 & -0.0034 & 0 & -0.0026 \\ -0.0155 & -0.0106 & -0.0043 & 0.0026 & 0 \end{bmatrix} \tag{3.2}$$

Results are shown in Tables 2, 3 and 4. For each objective function (minimization of CO_2, FCR and minimum distance), the solution is evaluated in the other two.

Table 2: Result when minimizing CO_2.

CO_2 (kg)	FCR (R$)	Distance (m)
344.884	15.339	31906.361
Route	0 – 3 – 4 – 2 – 1 – 0	

[3]https://cetesb.sp.gov.br/veicular/relatorios-e-publicacoes/

Table 3: Result when minimizing FCR.

CO_2 (kg)	FCR (R$)	Distance (m)
344.884	**15.339**	31906.361
Route	0 – 3 – 4 – 2 – 1 – 0	

Table 4: Results when minimizing distance.

CO_2 (kg)	FCR (R$)	Distance (m)
560.355	20.085	**31906.361**
Route	0 – 1 – 2 – 4 – 3 – 0	

Looking at Tables 2, 3 and 4, we note that two different routes were found for the three objective functions. Problems minimizing CO_2 and fuel consumption (FCR) provided the same solution, while for the minimum distance problem we have a different route, with the same optimal value in distance for all cases. To better understand these results, Tables 5 and 6 show the cost of each arc for both routes, rounded to three decimals.

Table 5: Route 0 – 3 – 4 – 2 – 1 – 0.

Arc	Distance (m)	FCR (R$)	CO_2 (kg)
0 – 3	10003.242	3.332	41.738
3 – 4	8116.461	3.605	74.196
4 – 2	7704.099	4.277	108.854
2 – 1	5427.043	3.615	104.394
1 – 0	655.515	0.510	15.702
	31906.361	15.339	344.884

The distance traveled on both routes is equal because the arcs traversed are symmetrical, for example (0 – 3) and (3 – 0), on routes 1 and 2 respectively. However, when we calculate the arc cost (1 – 2) and (2 – 1) for the other functions, these have different costs, and consequently, the route on Table 5 minimizes the costs of the other functions discussed here. If we only consider distance tough, both routes provide a minimal cost.

Looking at Table 6 it can be seen that routes start with smaller distances and end with greater ones, making both the FCR cost and CO_2 emissions higher, since they are proportional to truck weight. The route on Table 5 reverses the case: the longest path is traveled with less load.

In this small validation example, solutions for the FCR function and emission of CO_2 resulted in the same path, due to the size of the problem; in other cases, as seen next, different solutions were obtained.

Table 6: Route 0 – 1 – 2 – 4 – 3 – 0.

Arc	Distance (m)	FCR (R$)	CO_2 (kg)
0 – 1	655.515	0.219	2.765
1 – 2	5427.043	2.410	49.847
2 – 4	7704.099	4.277	110.155
4 – 3	8116.461	5.406	156.827
3 – 0	10003.242	7.773	240.761
	31906.361	20.085	560.355

3.2 Case Study: CORESO Cooperative

CORESO cooperative collects recyclable materials from some selected neighborhoods in the city of Sorocaba, state of São Paulo, Brazil, predominantly on the eastern part of the city. In 2016, it covered 230 streets (nodes) from Monday to Friday, comprising around 9000 collection points (door-to-door collection and collective generators). The collection points are illustrated[4] in Figure 3.

Routes were separated by weekdays, as requested by the cooperative administrative board, and demand requires two daily routes, for trucks with a capacity of 4000 kg. Tables 7, 8 and 9 present in **bold** the results obtained with the three objective functions previously described (CO_2, FCR and distance) respectively, and the number of nodes of each route, including the depot. The remaining columns show the values of the other two objective functions when calculated on the optimal route. The number of nodes sums up to 235 because some streets are visited more than once a week.

Table 7: Results for the cooperative when minimizing emissions of CO_2. For the sake of comparison, numbers in italics represent the solution when all slopes are considered zero. Although seemingly small, there are differences in four of five days.

	Distance m	FCR R$	**CO_2 kg**	*CO_2 kg (no slopes)*	Number of nodes
Monday	17932.364	6.694	**104.993**	*106.945*	60
Tuesday	10576.397	3.903	**61.394**	*61.394*	49
Wednesday	9606.025	3.516	**54.400**	*57.269*	46
Thursday	13257.440	4.920	**77.865**	*77.889*	39
Friday	9273.732	3.416	**53.613**	*53.561*	41
Totals	60645.958	22.449	**352.265**	*357.058*	235

Table 7 brings, for comparison purposes, the solution for the CO_2 emissions minimization problem, when all slopes are made zero (numbers in italics). For Monday, Wednesday and Thursday

[4] Figures 3 to 18 were made with a Python [9] script created by the authors. The script uses the GeoPandas library [14] with maps provided by OpenStreetMap [16].

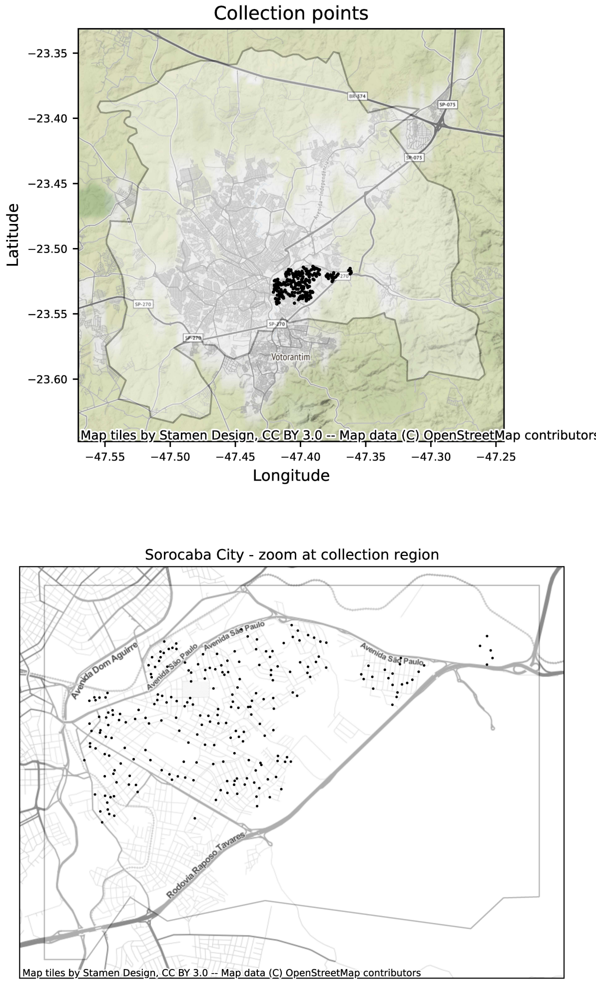


Figure 3: Collection points for recyclable materials at Sorocaba, state of São Paulo, Brazil. In the upper picture the points and the city boundaries can be seen. In the lower picture, there is a zoom comprising all points.

results considering the inclination are smaller, and thus better, than the zero slope ones. Although counterintuitive, that is one of the main results of this work. Clearly, larger slopes must result in larger CO_2 emissions, when sections of the route are taken individually. Equations (2.2) and (2.4) show that, when considering the whole route, inclination and weight (truck plus cargo) play together, resulting in sections with larger slopes coming first in the route, when the truck is almost empty, thus effectively reducing the emission. Tuesday reaches a match and Friday is a little worse.

Table 8: Results for the cooperative when minimizing consumption of fuel (FCR).

	Distance m	**FCR R$**	CO_2 kg	Number of nodes
Monday	17932.364	**6.644**	104.993	60
Tuesday	10572.502	**3.903**	61.407	49
Wednesday	9568.617	**3.510**	54.544	46
Thursday	13180.610	**4.902**	77.889	39
Friday	9148.581	**3.388**	53.723	41
Totals	60402.674	**22.347**	352.556	235

Table 9: Results for the cooperative when minimizing distance.

	Distance m	FCR R$	CO_2 kg	Number of nodes
Monday	**17932.364**	6.721	108.493	60
Tuesday	**10560.528**	4.069	69.564	49
Wednesday	**9490.884**	3.755	66.614	46
Thursday	**13094.096**	4.923	79.846	39
Friday	**9148.581**	3.388	53.723	41
Totals	**60226.453**	22.856	378.240	235

Table 7 (minimization of emissions) shows that the net reduction on the CO_2 emission is 4.793 kg per week, totaling almost 250 kg a year when slopes are taken into account. In the case of the fuel minimization function (results in Table 8), the decrease on the CO_2 emission is smaller (4.502 kg per week, 234.104 kg a year) when compared with the minimization of emissions with zero slope, as expected.

Minimization of emissions (Table 7) with slopes in play and minimization of fuel consumption (Table 8) have very close weekly emissions – a difference of 0.291 kg per week – with an added bonus of an economy of roughly R$ 5.30 a year. Of course the decision is up to the managerial board, but an extra reduction of emissions seems to be worth this extra cost.

Distance minimization (Table 9) can short the routes in 419 m per week or 21 km a year, with an expressive increase in CO_2 emissions (about 1350 kg a year). One can argue that shorter travels can extend the lifespan of the trucks and save time, but these savings are negligible: 21 km corresponds to only 0.67 % of the total distance traveled during a year.

To summarize, the minimization of CO_2 emissions can prevent 250 kg of pollutant reaching the atmosphere a year, while increasing fuel costs and distance traveled by a negligible amount of R$ 5.30 and 21 km, also per year respectively, a very small price to pay for the reduction of greenhouse gases and thus the overall improvement of environmental conditions.

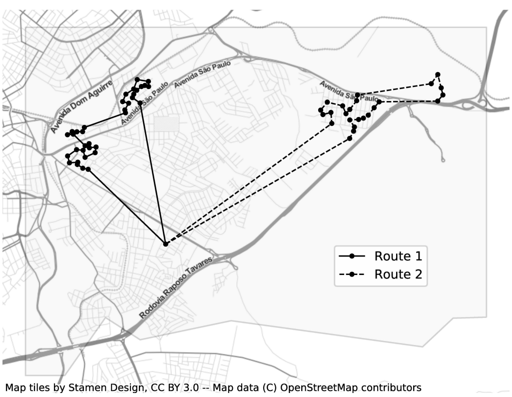
3.3 Results for Monday

Table 10 and Figures 4, 5 and 6 illustrate for Monday, results and the routes minimizing CO_2 emissions, FCR cost and distance, respectively. Table A.1 presents the streets and their respective indices.

Table 10: Summary of results for Monday. Numbers in **bold** are the minimum for that objective function.

Objective Function	Distance m	FCR R$	CO_2 kg	Number of nodes
CO_2 emission	17932.364	6.644	**104.993**	60
FCR	17932.364	**6.644**	104.993	60
Distance	**17932.364**	6.721	108.493	60

As it can be seen, routes for the minimization of CO_2 emissions and FCR cost are exactly the same, showing that inclinations probably were not big enough to make a difference. For distance minimization we have the same collection points, but in opposite directions, hence the same distance in all cases, but with an increase of 3.5 kg in emissions due to heavier loads close to the end of the route.



$0 \rightarrow 33 \rightarrow 29 \rightarrow 32 \rightarrow 38 \rightarrow 28$
$37 \leftarrow 35 \leftarrow 34 \leftarrow 1 \leftarrow 27 \leftarrow 39$
$36 \rightarrow 30 \rightarrow 2 \rightarrow 31 \rightarrow 44 \rightarrow 43$
$56 \leftarrow 54 \leftarrow 41 \leftarrow 40 \leftarrow 45 \leftarrow 42$
$57 \rightarrow 55 \rightarrow 53 \rightarrow 52 \rightarrow 58 \rightarrow 50$
$49 \leftarrow 48 \leftarrow 59 \leftarrow 51 \leftarrow 47 \leftarrow 46$

$0 \rightarrow 23 \rightarrow 9 \rightarrow 21 \rightarrow 8 \rightarrow 10$
$6 \leftarrow 20 \leftarrow 14 \leftarrow 22 \leftarrow 12 \leftarrow 11$
$7 \rightarrow 5 \rightarrow 4 \rightarrow 3 \rightarrow 19 \rightarrow 18$
$25 \leftarrow 24 \leftarrow 13 \leftarrow 15 \leftarrow 16 \leftarrow 17$
26

Figure 4: Routes minimizing CO_2 emissions for Monday. 60 collection points are covered, totaling a distance of 17932.364 m, 104.993 kg of emitted CO_2 and a FCR cost of R$ 6.694. Numbers in routes refer to Table A.1.

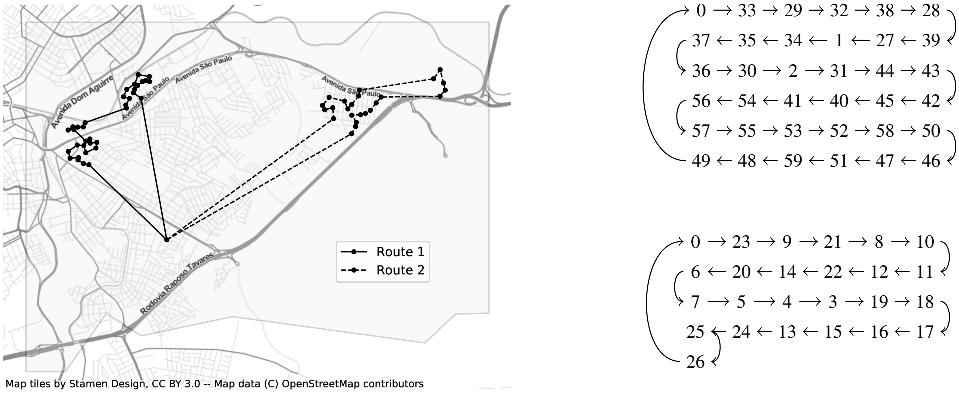


Figure 5: Routes minimizing FCR for Monday. 60 collection points are covered, totaling a distance of 17932.364 m, 104.993 kg of emitted CO_2 and a FCR cost of R$ 6.644. Numbers in nodes refer to Table A.1.

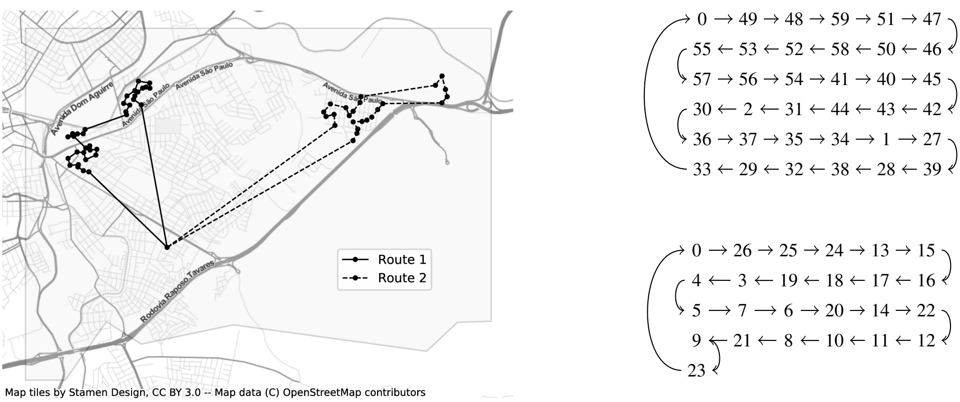


Figure 6: Routes minimizing distance for Monday. 60 collection points are covered, totaling a distance of 17932.364 m, 108.493 kg of emitted CO_2 and a FCR cost of R$ 6.721. Numbers in nodes refer to Table A.1.

3.4 Results for Tuesday

Table 11 and Figures 7, 8 and 9 illustrate for Tuesday, results and routes minimizing CO_2 emissions, FCR cost and distance, respectively. Table A.2 presents the streets and their respective indices.

Table 11: Summary of results for Tuesday. Numbers in **bold** are the minimum for that objective function.

Objective Function	Distance m	FCR R$	CO_2 kg	Number of nodes
CO_2 emission	10576.397	3.903	**61.394**	49
FCR	10572.502	**3.903**	61.407	49
Distance	**10560.528**	4.069	69.564	49

For Tuesday results for emission and FCR minimization are very close, as well as their respective routes (only a few streets in different places along the route). On the other hand, distance minimization was able to cut off only 15.869 m with a corresponding increase of 8.17 kg of CO_2, clearly showing that a few tweaks in the routes can have a big impact on the amount of emission, with minimal effect on the distance.

It is interesting to notice that on the distance minimization problem, there is a clear unbalance on the route size, as if the solution is built by trying to travel as much as possible in one route (possibly through shorter sections), leaving a few streets for the second.

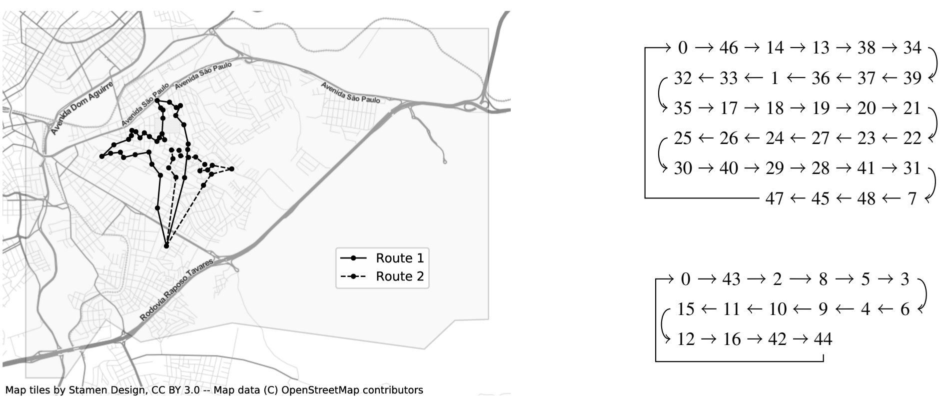
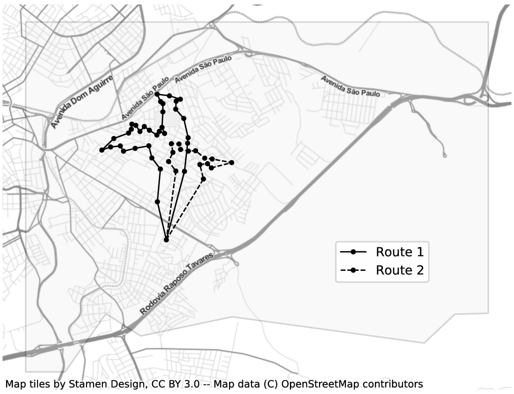


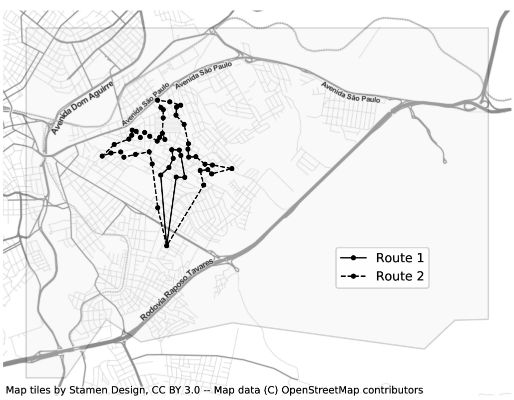
Figure 7: Routes minimizing CO_2 emissions for Tuesday. 49 collection points are covered, totaling a distance of 10576.397 m, 61.394 kg of emitted CO_2 and a FCR cost of R$ 3.903. Numbers in nodes refer to Table A.2.



0 → 46 → 14 → 13 → 38 → 34
32 ← 33 ← 1 ← 36 ← 39 ← 37
35 → 17 → 18 → 19 → 20 → 21
25 ← 26 ← 24 ← 27 ← 23 ← 22
30 → 40 → 29 → 28 → 41 → 31
47 ← 45 ← 48 ← 7

0 → 43 → 6 → 3 → 2 → 8
15 ← 11 ← 10 ← 9 ← 4 ← 5
12 → 16 → 42 → 44

Figure 8: Routes minimizing FCR for Tuesday. 49 collection points are covered, totaling a distance of 10572.502 m, 61.407 kg of emitted CO_2 and a FCR cost of R$ 3.903. Numbers in nodes refer to Table A.2.



0 → 45 → 42 → 16 → 12 → 15
44 ← 46 ← 11

0 → 47 → 48 → 7 → 31 → 41
26 ← 25 ← 30 ← 40 ← 29 ← 28
24 → 27 → 23 → 22 → 21 → 20
33 ← 32 ← 35 ← 17 ← 18 ← 19
1 → 36 → 39 → 37 → 34 → 38
5 ← 4 ← 9 ← 10 ← 14 ← 13
8 → 2 → 3 → 6 → 43

Figure 9: Routes minimizing distance for Tuesday. 49 collection points are covered, totaling a distance of 10560.528 m, 69.564 kg of emitted CO_2 and a FCR cost of R$ 4.069. Numbers in nodes refer to Table A.2.

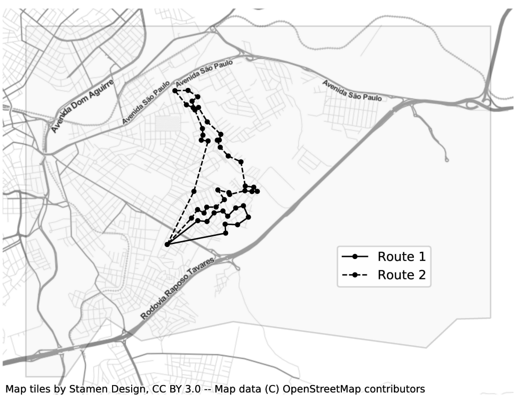
3.5 Results for Wednesday

Table 12 and Figures 10, 11 and 12 illustrate for Wednesday, results and routes minimizing CO_2 emissions, FCR cost and distance, respectively. Table A.3 presents the streets and their respective indices.

Table 12: Summary of results for Wednesday. Numbers in **bold** are the minimum for that objective function.

Objective Function	Distance m	FCR R$	CO_2 kg	Number of nodes
CO_2 emission	9606.025	3.516	**54.400**	46
FCR	9568.617	**3.510**	54.544	46
Distance	**9490.884**	3.755	66.614	46

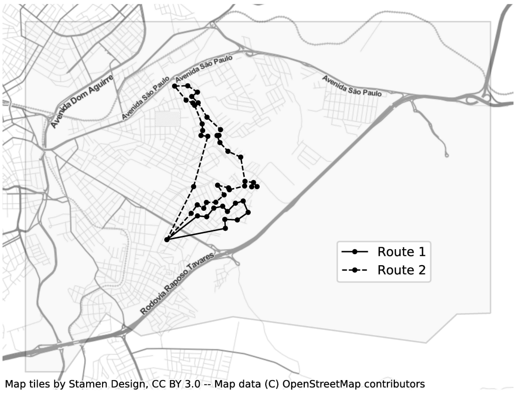
Once more minimization of emissions and FCR have very similar routes, with a negligible difference on the CO_2 emission: only 0.144 kg. On the other hand, Wednesday has the biggest difference in emission when comparing with the distance minimization function, 12.214 kg, for a route 115.141 m shorter. Environmental-wise, it makes much more sense to choose the minimum emission route.



$0 \to 41 \to 28 \to 42 \to 30 \to 27$
$44 \leftarrow 38 \leftarrow 35 \leftarrow 40 \leftarrow 39 \leftarrow 29$

$0 \to 31 \to 21 \to 9 \to 20 \to 10$
$13 \leftarrow 19 \leftarrow 17 \leftarrow 14 \leftarrow 15 \leftarrow 16$
$12 \to 18 \to 11 \to 7 \to 6 \to 8$
$25 \leftarrow 24 \leftarrow 1 \leftarrow 4 \leftarrow 22 \leftarrow 5$
$26 \to 23 \to 2 \to 3 \to 32 \to 33$
$43 \leftarrow 45 \leftarrow 37 \leftarrow 36 \leftarrow 34$

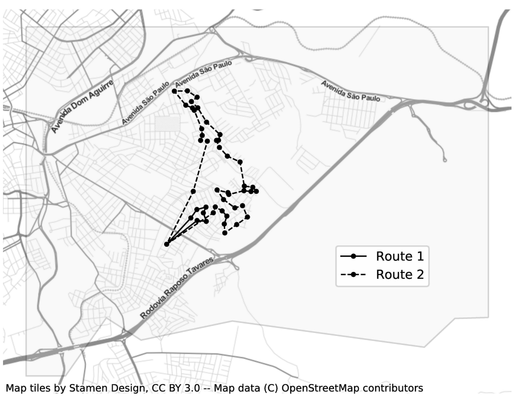
Figure 10: Routes minimizing CO_2 emissions for Wednesday. 46 collection points are covered, totaling a distance of 9606.025 m, 54.400 kg of emitted CO_2 and a FCR cost of R$ 3.516. Numbers in nodes refer to Table A.3.



$0 \rightarrow 41 \rightarrow 28 \rightarrow 42 \rightarrow 30 \rightarrow 27$
$44 \leftarrow 38 \leftarrow 35 \leftarrow 40 \leftarrow 39 \leftarrow 29$

$0 \rightarrow 31 \rightarrow 21 \rightarrow 9 \rightarrow 20 \rightarrow 10$
$14 \leftarrow 17 \leftarrow 19 \leftarrow 13 \leftarrow 12 \leftarrow 16$
$15 \rightarrow 18 \rightarrow 11 \rightarrow 7 \longrightarrow 6 \longrightarrow 8$
$25 \leftarrow 24 \leftarrow 1 \longleftarrow 4 \leftarrow 22 \leftarrow 5$
$26 \rightarrow 23 \longrightarrow 2 \longrightarrow 3 \rightarrow 32 \rightarrow 33$
$43 \leftarrow 45 \leftarrow 37 \leftarrow 36 \leftarrow 34$

Figure 11: Routes minimizing FCR for Wednesday. 46 collection points are covered, totaling a distance of 9568.617 m, 54.544 kg of emitted CO_2 and a FCR cost of R$ 3.510. Numbers in nodes refer to Table A.3.



$0 \rightarrow 44 \rightarrow 38 \rightarrow 37 \rightarrow 36 \rightarrow 45$
43

$0 \rightarrow 35 \rightarrow 34 \rightarrow 40 \rightarrow 39 \rightarrow 28$
$33 \leftarrow 29 \leftarrow 27 \leftarrow 30 \leftarrow 42 \leftarrow 41$
$32 \rightarrow 3 \longrightarrow 2 \rightarrow 23 \rightarrow 26 \rightarrow 25$
$8 \longleftarrow 5 \leftarrow 22 \leftarrow 4 \longleftarrow 1 \leftarrow 24$
$6 \longrightarrow 7 \rightarrow 11 \rightarrow 18 \rightarrow 15 \rightarrow 14$
$10 \leftarrow 16 \leftarrow 12 \leftarrow 13 \leftarrow 19 \leftarrow 17$
$20 \rightarrow 9 \rightarrow 21 \rightarrow 31$

Figure 12: Routes minimizing distance for Wednesday. 46 collection points are covered, totaling a distance of 9490.884 m, 66.614 kg of emitted CO_2 and a FCR cost of R$ 3.755. Numbers in nodes refer to Table A.3.

3.6 Results for Thursday

Table 13 and Figures 13, 14 and 15 illustrate for Thursday, results and routes minimizing CO_2 emissions, FCR cost and distance, respectively. Table A.4 presents the streets and their respective indices.

Table 13: Summary of results for Thursday. Numbers in **bold** are the minimum for that objective function.

Objective Function	Distance m	FCR R$	CO_2 kg	Number of nodes
CO_2 emission	13257.440	4.920	**77.865**	39
FCR	13180.610	**4.902**	77.889	39
Distance	**13094.096**	4.923	79.846	39

Thursday is perhaps the most uniform day, with very close results for all objective functions, maybe because it has the smallest number of streets in the week. Despite that, emissions and FCR minimization have very different routes when compared to the previous days. Emissions and distance are not that different either, with a reduction of 1.981 kg of CO_2 for an extra 163.344 m.

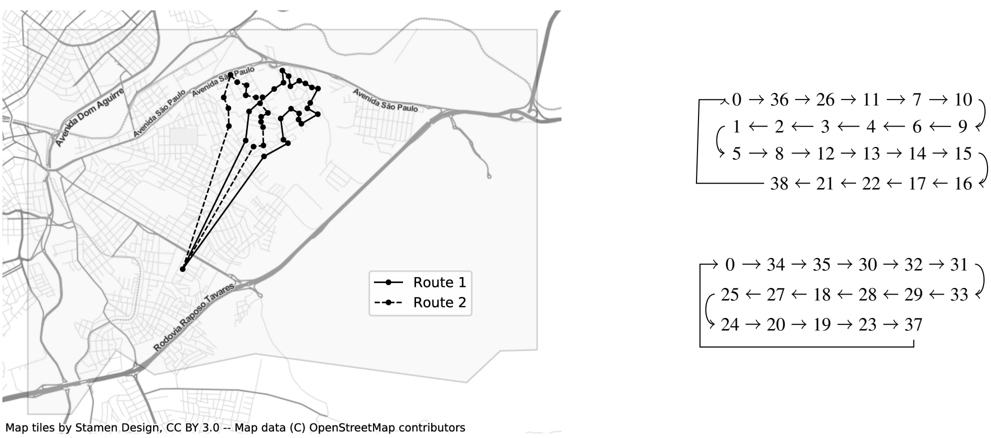


Figure 13: Routes minimizing CO_2 emissions for Thursday. 46 collection points are covered, totaling a distance of 9606.025 m, 54.400 kg of emitted CO_2 and a FCR cost of R$ 3.516. Numbers in nodes refer to Table A.4.

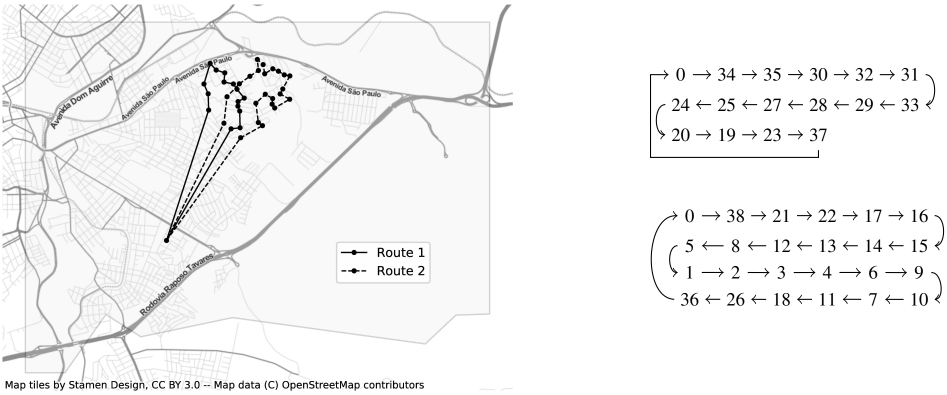


Figure 14: Routes minimizing FCR for Thursday. 46 collection points are covered, totaling a distance of 9568.617 m, 54.544 kg of emitted CO_2 and a FCR cost of R$ 3.510. Numbers in nodes refer to Table A.4.

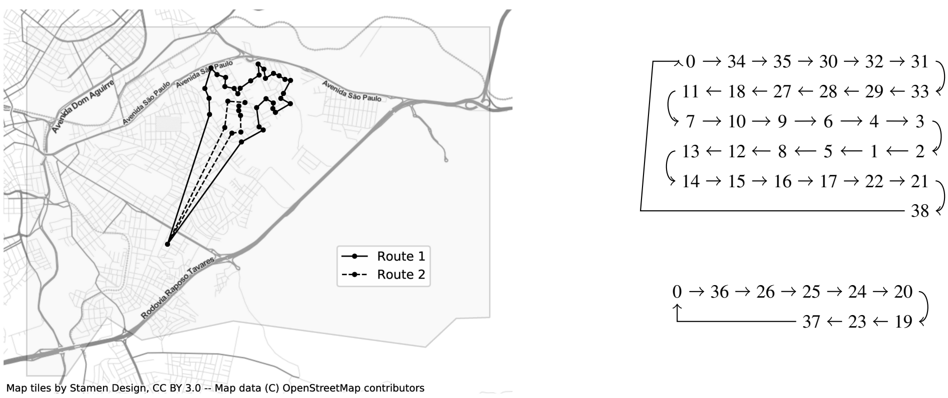


Figure 15: Routes minimizing distance for Thursday. 46 collection points are covered, totaling a distance of 9490.884 m, 66.614 kg of emitted CO_2 and a FCR cost of R$ 3.755. Numbers in nodes refer to Table A.4.

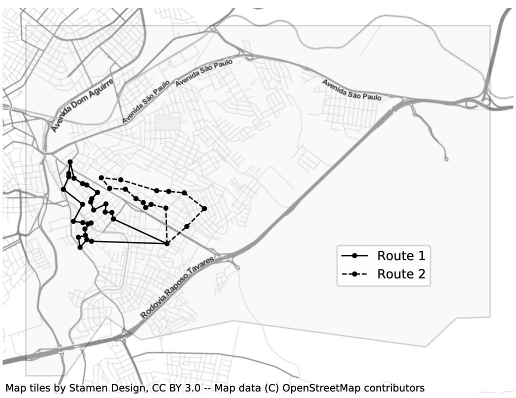
3.7 Results for Friday

Table 14 and Figures 16, 17 and 18 illustrate for Friday, the routes minimizing CO_2 emissions, FCR cost and distance, respectively. Table A.5 presents the streets and their respective indices.

Table 14: Summary of results for Friday. Numbers in **bold** are the minimum for that objective function.

Objective Function	Distance m	FCR R$	CO_2 kg	Number of nodes
CO_2 emission	9273.732	3.416	**53.613**	41
FCR	9148.581	**3.388**	53.723	41
Distance	**9148.581**	3.388	53.723	41

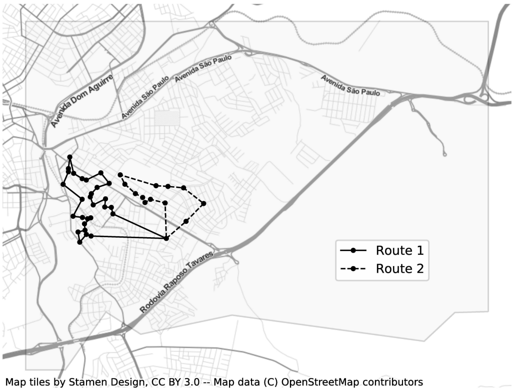
Finally, Friday also has very uniform results, with FCR and distance minimization giving the same routes, with little gain when compared to CO_2 emission minimization (only 0.110 kg, the smallest amongst all days).



0 → 18 → 21 → 15 → 22 → 17
35 ← 16 ← 23 ← 19 ← 38 ← 20
27 → 26 → 25 → 24 → 28 → 30
36 ← 29 ← 34 ← 32 ← 33 ← 31
37 → 39 → 40

0 → 2 → 3 → 7 → 8 → 6
11 ← 12 ← 4 ← 13 ← 14 ← 10
1 → 9 → 5

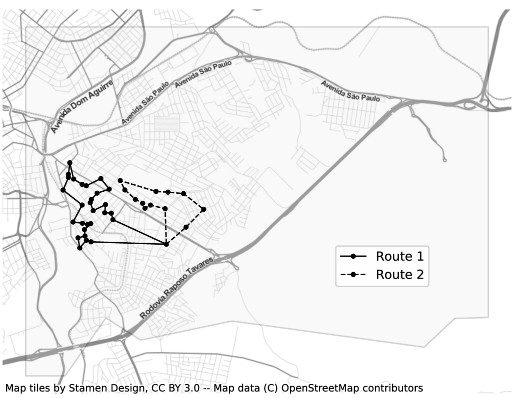
Figure 16: Routes minimizing CO_2 emissions for Friday. 46 collection points are covered, totaling a distance of 9606.025 m, 54.400 kg of emitted CO_2 and a FCR cost of R$ 3.516. Numbers in nodes refer to Table A.5.



$0 \to 18 \to 21 \to 15 \to 22 \to 17$
$35 \leftarrow 16 \leftarrow 23 \leftarrow 19 \leftarrow 38 \leftarrow 20$
$27 \to 26 \to 25 \to 24 \to 28 \to 30$
$34 \leftarrow 32 \leftarrow 33 \leftarrow 13 \leftarrow 14 \leftarrow 31$
$29 \to 36 \to 37 \to 39 \to 40$

$0 \to 2 \to 3 \to 7 \to 8 \to 6$
$9 \leftarrow 1 \leftarrow 11 \leftarrow 12 \leftarrow 4 \leftarrow 10$
5

Figure 17: Routes minimizing FCR for Friday. 46 collection points are covered, totaling a distance of 9568.617 m, 54.544 kg of emitted CO_2 and a FCR cost of R$ 3.510. Numbers in nodes refer to Table A.5.



$0 \to 18 \to 21 \to 15 \to 22 \to 17$
$35 \leftarrow 16 \leftarrow 23 \leftarrow 19 \leftarrow 38 \leftarrow 20$
$27 \to 26 \to 25 \to 24 \to 28 \to 30$
$34 \leftarrow 32 \leftarrow 33 \leftarrow 13 \leftarrow 14 \leftarrow 31$
$29 \to 36 \to 37 \to 39 \to 40$

$0 \to 2 \to 3 \to 7 \to 8 \to 6$
$9 \leftarrow 1 \leftarrow 11 \leftarrow 12 \leftarrow 4 \leftarrow 10$
5

Figure 18: Routes minimizing distance for Friday. 46 collection points are covered, totaling a distance of 9490.884 m, 66.614 kg of emitted CO_2 and a FCR cost of R$ 3.755. Numbers in nodes refer to Table A.5.

The biggest challenge was to implement and test Function (2.4). The inclusion of Function (2.13) and distance was used to compare results. We note that, with the routes obtained, the results are quite satisfactory in the sense that, even running paths with greater distances, still we have gained in the reduction of emission CO_2 and fuel consumption, which benefits the environment in addition to generating a smaller workforce for the vehicle, increasing its useful life.

4 FINAL CONSIDERATIONS

In this work we have presented the application of a Capacited Vehicle Routing Problem (CVRP) for a recycling cooperative at Sorocaba, state of São Paulo, Brazil, comparing three different and independent objectives: minimization of either distance, or fuel consumption rate or CO_2 emissions. For the latter the inclination of each section of the route is considered, which affects the emission rate.

The streets were previously chosen by the cooperative managerial board and grouped by business days. Two trucks perform the collection during the week simultaneously, meaning that for each objective two routes are obtained.

Results have shown that is possible, with minor changes in the routes already in practice by the cooperative, emit a considerable smaller amount of CO_2 in the atmosphere in the course of a year, with negligible increase either in traveled distance or fuel cost. It should be stressed that these minor adjustments are more easily adopted by both the board and the workers because they are associated with smaller business logic distress.

Giving the competing nature of the objectives here addressed, CO_2 emission, distance and fuel consumption rate, further research points to multiobjective formulations. A bi-objective problem – minimization of both CO_2 and distance – is being prospectively addressed by the authors.

Investment in green routes is necessary since environmental problems caused by pollutant emissions from motor vehicles is one of the biggest factors of air pollution and consequently of climate change. With the increase in the fleet of vehicles, alternative fuels are increasingly becoming the focus of research. However, they cannot replace fossil fuels yet as they are energetically less efficient, thus proactively seeking better routes while keeping costs in check presents itself as a very viable option.

Acknowledgments

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APPENDIX A: COLLECTION POINTS TABLES

Table A.1: Collection points for Monday.

Index	Address	Index	Address
1	Condomínio Cruzeiro do Sul	31	Rua Cruz e Souza
2	Condomínio Torre de Vicenza	32	Rua Gonçalo Vecina de la Vina
3	Rua João Tiburcio dos Santos	33	Rua João Ferreira da Silva
4	Rua Miguel Sayeg	34	Rua Antonio Gomes Morgado
5	Rua Celina Stela Corradi Beu	35	Rua São Miguel Arcanjo
6	Rua Adone Sotovia	36	Rua Ana Nery
7	Rua Nagib Jorge Murad	37	Rua Martins França
8	Rua José Maria Christ	38	Rua João Frederico Hingst
9	Rua Ismael Estanislau de Arruda	39	Rua Doutor Nicolau Alonso Martins
10	Rua Josephina Rodrigues Colo	40	Rua Pedro Jacob
11	Rua João Martinez	41	Rua Tobias Barreto
12	Rua Florencio Vieira da Rocha	42	Rua Francisco de Paula Aquino
13	Rua Agripino Guedes	43	Rua D'Abreu Medeiros
14	Rua Luiz Celestino Bertanha	44	Rua Rafael Laino
15	Rua Guido Mencacci	45	Rua General Antunes Gurjão
16	Rua Epaminondas Neves	46	Rua Claudio Furquim
17	Rua Antonio Martins Caixeiro Soriano	47	Rua Doutor Ruy Barbosa
18	Rua Laila Gallep Sacker	48	Rua Coronel Jose Tavares
19	Rua Milton Ribeiro Pinto	49	Rua Pericles Pilar
20	Rua Fernando Silva	50	Rua Constantino Senger
21	Rua Irineu Momesso	51	Rua Isaac Pacheco
22	Rua Benedito Barbosa	52	Rua Ataliba Borges
23	Rua Antonio Guilherme da Silva	53	Rua Luiz Paes de Almeida
24	Rua Sargento Paulino Claro dos Santos	54	Rua Felipe Betti
25	Rua Dom Paulo Rolim Loureiro	55	Rua Visconde de Mauá
26	Rua Amalia Fernandes Rodrigues	56	Rua Joaquim Bastos
27	Rua dos Expedicionarios	57	Rua Vital Brasil
28	Rua Maria Garcia Alcolea	58	Rua Voluntario Altino
29	Rua Doutor Delfim Moreira	59	Rua Gustavo Schrepel
30	Rua Padre Lessa		

Table A.2: Collection points for Tuesday.

Index	Address	Index	Address
1	Rua Luigi Brunetti	25	Rua Cesario de Aguiar
2	Rua Afonso Gabriotti	26	Rua Gioto Pannunzio
3	Rua Jornalista Hernani Pereira	27	Rua Almeida Falcão
4	Rua Jorge Caracante	28	Rua Paulo Eiro
5	Avenida Dom Pedro I	29	Rua Frei Eugênio Becker
6	Rua Pindorama	30	Rua Professor Eneas Proença de Arruda
7	Rua Pedro José Senger	31	Rua Maria Aparecida Brunetti
8	Rua Ramon Haro Martini	32	Rua Estacio de Sa
9	Rua Gastão Vidigal	33	Rua Doutor Alfredo Maia
10	Rua Olímpio Loureiro	34	Rua Teotonio de Araujo
11	Rua Pedro Nolasco de Campos	35	Rua Benjamin dos Santos
12	Rua Aristide da Silva Lobo	36	Rua Antonio Monteiro
13	Rua General Argolo	37	Rua Rodrigues do Prado
14	Rua Rodrigues de Mello	38	Rua Margarida Izar
15	Rua João Nobrega de Almeida	39	Avenida José Benedito de Lima
16	Rua Hipólito José da Costa	40	Rua Wilson Fusco
17	Rua José do Patrocínio	41	Rua Jorge Courbassier
18	Rua Doutor Emílio Ribas	42	Rua Guilherme Marconi
19	Rua Lopes Trovão	43	Rua Thadeu Grembecki
20	Rua Joaquim Pires	44	Rua Padre Pedro Domingos Paes
21	Rua Conselheiro Antonio Prado	45	Rua Tiburcio Gabriel Torres Monteiro
22	Rua Martins de Oliveira	46	Rua Pedro Acacio de Marcos
23	Rua Professor Fonseca Junior	47	Rua Fernando Luiz Grohman
24	Rua Americo Brasiliense	48	Rua Antonia Lopes Bravo

Table A.3: Collection points for Wednesday.

Index	Address	Index	Address
1	Rua Joana Decaria Tota	24	Rua Pedro Geremias Alves
2	Rua Ramon Haro Martini	25	Rua Vicente Celestino
3	Rua Pedro Jose Senger	26	Rua Joana Decaria Totta
4	Rua Hosmar Dahir	27	Rua Agostinho Decaria
5	Rua Antonio Aidar	28	Rua Vicente Decaria
6	Rua Antonio Arrojo Peres	29	Rua Robina Cacielo Decaria
7	Rua Joao Delgado Hidalgo	30	Rua Coronel Paulo Foot Guimarães
8	Rua Dirceu D'Almeida	31	Rua João Valentino Joel
9	Rua Carmem Galan Archilla	32	Rua Mario Piccini
10	Rua Pedro Sunica Neto	33	Rua Paschoal Bernal Vecina
11	Rua Agustinho de Vito	34	Rua Jose Prestes de Barros
12	Rua Doutor Gabriel Rezende Passos	35	Rua Jose Bonadia
13	Rua Adolfo Grizzi Santos	36	Rua Ivan Santos Albuquerque
14	Rua Pedro Peres	37	Rua Hortencio Piaya Martinez
15	Rua Jose Balera	38	Rua Antonio Antunes de Almeida
16	Rua Umberto Ferro	39	Rua Ambrosina do Amaral Marchetti
17	Rua Luiz Vicente Verlangieri	40	Rua Manuel Ribeiro de Andrade
18	Rua Sizina Azevedo Schrepel	41	Rua Pedro Teodoro de Almeida
19	Rua Pedro de Goes	42	Rua Wilma Tavares Simoni
20	Rua Professor Dorival Dias de Carvalho	43	Avenida Carlos Sonetti
21	Rua Renato Swensson	44	Rua Bayard Nobrega de Almeida
22	Rua Domenico Matteis	45	Rua Emerenciano Prestes de Barros
23	Rua Joaquim Scherepel		

Table A.4: Collection points for Thursday.

Index	Address	Index	Address
1	Rua Dionisio Reis dos Santos	20	Rua Jose Roberto Moncayo
2	Rua Jose de Oliveira	21	Rua Plinio de Almeida
3	Rua Benedito de Campos	22	Rua Professor Nelson Guedes
4	Rua Fernando Martins Costa	23	Rua Bernardo Martins Junior
5	Rua Dorothy de Oliveira	24	Rua Lucimara Godoy Zambonini
6	Rua Jose Rosa	25	Rua Carmem Ruiz Moncayo
7	Rua Eugenio Leite	26	Rua Lauro Alves Lima
8	Rua Eduardo Sandano	27	Rua Jose Martinez Y. Martinez
9	Rua Solange Victoretti	28	Rua Luigi Lava Melapague
10	Rua Antonio Rodrigues Sanches	29	Rua Artur Tarsitani
11	Rua Mario Guilherme Notari	30	Rua Santos Severo Scapol
12	Rua Major Barros França	31	Rua Francisco Mucciolo
13	Rua João Batista de Moraes	32	Rua Humberto Notari
14	Rua Renato Lucci	33	Euclides Medeiros
15	Rua Antonia Camargo Nunes	34	Belmira Loireiro de Almeida
16	Rua Florencio Antonio Pires	35	Rua Demercindo Alves da Silva
17	Rua Joao Moncayo	36	Rua Plínio Miguel
18	Alameda Professor Horácio Ribeiro	37	Rua Joao Augusto Gomes
19	Rua Jose Del Cistia	38	Rua Rubesval Luiz Jose

Table A.5: Collection points for Friday.

Index	Address	Index	Address
1	Rua Comandante Salgado	21	Rua Capitao Padilha de Camargo
2	Rua Andre Matiello	22	Rua Doutor Alvaro Guião
3	Rua Aristeu Prestes de Barros	23	Rua Vidal de Negreiros
4	Rua Fernao Salles	24	Rua Doutor Ruy Barbosa
5	Rua Joaquim Rodrigues de Barros	25	Rua Jose Martins
6	Rua Jeronimo Antonio Fiuza	26	Rua Duarte da Costa
7	Rua Joao Valentino Joel	27	Rua Newton Prado
8	Rua Fernando Luiz Grohman	28	Rua Santa Maria
9	Rua Proessor Luiz de Campos	29	Rua Manoel Lopes
10	Rua Teodoro Kaizel	30	Rua Doutor Oliverio Pilar
11	Rua Nhozinho Prestes	31	Rua Francisco Glicerio
12	Rua Marquesa de Santos	32	Rua Tereza Lopes
13	Rua Assis Machado	33	Rua Barcelona
14	Rua Quinzinho de Barros	34	Rua Sa Fleury
15	Rua Doutor Campos Salles	35	Rua Sargento Antônio Remio Ribeiro
16	Rua Felipe Camarão	36	Rua Sevilha
17	Rua Raposo Tavares	37	Rua Madrid
18	Rua Augusto de Assis	38	Rua Thome de Souza
19	Rua Doutor Moreira Salles	39	Rua Catalunha
20	Rua Ricardo Severo	40	Rua Granada